

## Charged Higgs on $B^- \rightarrow \tau \bar{\nu}_\tau$ and $\bar{B} \rightarrow P(V) \ell \bar{\nu}_\ell$

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**ABSTRACT:** We study the charged Higgs effects on the decays of  $B^- \rightarrow \tau \bar{\nu}_\tau$  and  $\bar{B} \rightarrow P(V) \ell \bar{\nu}_\ell$  with  $P = \pi^+, D^+$  and  $V = \rho^+, D^{*+}$ . We concentrate on the minimal supersymmetric standard model with nonholomorphic terms at a large  $\tan \beta$ . To extract new physics contributions, we define several physical quantities related to the decay rate and angular distributions to reduce uncertainties from the QCD as well as the CKM elements. With the constraints from the recent measurement on the decay branching ratio of  $B^- \rightarrow \tau \bar{\nu}_\tau$ , we find that the charged Higgs effects could be large and measurable.

**KEYWORDS:** Supersymmetric Standard Model, B-Physics, Beyond Standard Model.

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## 1. Introduction

Many exclusive hadronic  $B$  decay modes have been observed in branching ratios (BRs) and CP asymmetries (CPAs) at  $B$  factories [1]. However, it is hard to give conclusive theoretical predictions for most of the processes in the standard model (SM) due to the nonperturbative QCD effects. Consequently, it is not easy to tell whether there are some derivations between theoretical predictions and experimental measurements. To search for new physics, it is important to look for some observables which contain less theoretical uncertainties. With enormous  $B$  events, recently, the BELLE [2] and BABAR [3] Collaborations have measured the purely leptonic decay of  $B^- \rightarrow \tau \bar{\nu}_\tau$  as [4]

$$\begin{aligned}
 BR(B^- \rightarrow \tau \bar{\nu}_\tau) &= (1.79_{-0.49-0.46}^{+0.56+0.39}) \times 10^{-4} \text{ (BELLE)} \\
 &= (0.88_{-0.67}^{+0.68} \pm 0.11) \times 10^{-4} < 1.8 \times 10^{-4} \text{ (90\% C.L.) (BABAR)} \\
 &= (1.36 \pm 0.48) \times 10^{-4} \text{ (BELLE + BABAR)} \tag{1.1}
 \end{aligned}$$

This observation provides a possibility to detect new physics. It is well known that the SM contribution to the decay branching ratio arises from the charged weak interactions with the main uncertainty from the Cabibbo-Kobayashi-Maskawa (CKM) matrix element  $V_{ub}$  and the B-meson decay constant  $f_B$ . The value of  $V_{ub}$  has been constrained by the inclusive and exclusive charmless semileptonic  $B$  decays, given by  $|V_{ub}| = (4.39 \pm 0.33) \times 10^{-3}$  [1] and  $|V_{ub}| = (3.67 \pm 0.47) \times 10^{-3}$  [5], respectively. Obviously, when  $|V_{ub}|$  is fixed, the decay

of  $B^- \rightarrow \tau \bar{\nu}_\tau$  could be used to determine  $f_B$ , which should not be far away from that calculated by the lattice QCD [6] as well as extracted from other experimental data, such as  $\Delta M_B$  [5, 7]. Clearly, if there appears some significant derivation, it could imply the existence of physics beyond the SM.

The most interesting new physics contribution to the decay is the charged Higgs effect at tree level [8, 9]. Similar effect has also been studied in the inclusive [10] and exclusive [11] semileptonic  $B$  decays. It is known that the charged Higgs boson exists in any model with two or more Higgs doublets, such as the minimal supersymmetric standard model (MSSM) which contains two Higgs doublets  $H_d$  and  $H_u$  coupling to down and up type quarks, respectively. In the MSSM, it is natural to avoid the flavor changing neutral current (FCNC) at tree level. However, due to supersymmetric breaking effects, it is found that in the large  $\tan\beta$  region, the contribution to the down type quark masses from the nonholomorphic terms  $QD^c H_u$  generated at one-loop could be as large as that from the holomorphic ones  $QD_c H_d$  [12, 13]. Subsequently, many interesting Higgs related phenomena have been studied [14–18].

In this paper, we will study the charged Higgs contributions with the nonholomorphic corrections to the leptonic decays of  $B^- \rightarrow \ell \bar{\nu}_\ell$  and the exclusive semileptonic decays of  $B \rightarrow P(V)\ell \bar{\nu}_\ell$  where  $\ell$  denote as the charged leptons and  $P$  ( $V$ ) stand for the pseudoscalar (vector) mesons. In particular, we will investigate the differential decay rates and the lepton angular distributions in the exclusive semileptonic modes to examine the charged Higgs effects based on the constraint from the measurement on  $BR(B^- \rightarrow \tau \bar{\nu}_\tau)$ .

The paper is organized as follows. In section 2, we derive the couplings of charged Higgs to quarks by including the one-loop corrections to the Yukawa sector. In section III, we present the formalisms for the decay rates of  $B^- \rightarrow \ell \bar{\nu}_\ell$ , the differential decay rates and angular asymmetries of  $B \rightarrow P(V)\ell \bar{\nu}_\ell$  in the presence of the charged Higgs contributions. We display the numerical analysis in section 4. Finally, we summarize the results in section 5.

## 2. Couplings of charged Higgs to quarks

In models with two Higgs doublets, the general Yukawa couplings with radiative corrections for the quark sector under the gauge groups  $SU(2)_L \times U(1)_Y$  can be written as [17]

$$-\mathcal{L}_Y = \bar{Q}_L \left[ \mathbf{H}_d + \left( \epsilon_0 + \epsilon_Y Y_u Y_u^\dagger \right) \tilde{\mathbf{H}}_u \right] \mathbf{Y}_d D_R + h.c. \quad (2.1)$$

where  $Q_L^T = (U, D)_L$  and  $D_R$  denote the  $SU(2)$  doublet and singlet of quarks, respectively,  $\mathbf{H}_d^T = (\phi_d^+, \phi_d^0)$  and  $\tilde{\mathbf{H}}_u = -i\tau_2 \mathbf{H}_u^*$  with  $\mathbf{H}_u^T = (\phi_u^{*0}, -\phi_u^-)$  are the two Higgs doublets,  $\mathbf{Y}_{d(u)}$  is the  $3 \times 3$  Yukawa mass matrix for down (up) type quarks, and  $\epsilon_{0,Y}$  stand for the effects of radiative corrections. Since only the down-type quark mass matrix can have large radiative corrections, we will not address the parts related to  $\bar{Q}_L U_R$ . Moreover, for simplicity, we choose  $\mathbf{Y}_d$  to be a diagonal matrix  $\mathbf{Y}_{dij} = y_{di} \delta_{ij}$  while  $\mathbf{Y}_u$  is diagonalized by  $V_U^{0L} \mathbf{Y}_u V_U^{0R\dagger} \equiv U = \text{diag}\{y_u, y_c, y_t\}$  with  $V_U^{0L(R)}$  being unitary matrices. In terms of the charged weak interaction, denoted by  $I_W = \bar{U}_L \gamma^\mu D_L W_\mu^+$ , the Cabibbo-Kobayashi-Maskawa (CKM) matrix is  $V^0 = V_U^{0L}$ . From eq. (2.1), we know that due to the appearance

of  $\epsilon_Y$ , the down-type quark mass matrix, expressed by

$$\begin{aligned} M_D &= \left[ 1 + \tan \beta \left( \epsilon_0 + \epsilon_Y V^{0\dagger} U U^\dagger V^0 \right) \right] \mathbf{Y}_d v_d, \\ &= M_D^{\text{dia}} + \delta M_D, \end{aligned} \quad (2.2)$$

is no longer diagonal, where

$$\begin{aligned} M_{Di}^{\text{dia}} &= y_{di} v_d [1 + \tan \beta \epsilon_i], \\ \delta M_{Dij} &= y_{dj} v_d \tan \beta \epsilon_Y y_i^2 V_{tij}^0, \end{aligned} \quad (2.3)$$

with  $v_{d(u)} = \langle \phi_{d(u)}^0 \rangle$ ,  $\tan \beta = v_u/v_d$ ,  $V_{tij}^0 = V_{ti}^{0*} V_{tj}^0$  and  $\epsilon_i = \epsilon_0 + \epsilon_Y y_i^2 \delta_{i3}$ . Here we have neglected the contributions of  $y_{u(c)}$  due to the hierarchy  $y_u \ll y_c \ll y_t$ .

In order to diagonalize the mass matrix of eq. (2.2), we need to introduce new unitary matrices  $V_D^{L(R)}$  so that the physical states are given by

$$d_L = V_D^L D_L, \quad d_R = V_D^R D_R, \quad (2.4)$$

and the diagonalized mass matrix is  $m_D = V_D^L M_D V_D^{R\dagger}$ . Subsequently, we have the relationships

$$\begin{aligned} m_D m_D^\dagger &= V_D^L M_D M_D^\dagger V_D^{L\dagger}, \\ m_D^\dagger m_D &= V_D^R M_D^\dagger M_D V_D^{R\dagger}. \end{aligned} \quad (2.5)$$

Since the off-diagonal terms in eq. (2.2) are associated with  $\epsilon_Y$  which is much less than unity, we can find  $V_D^{L(R)}$  by the perturbation in  $\epsilon_Y$ . At the leading  $\epsilon_Y$ , the unitary matrices could be expressed by  $V_D^L \approx 1 + \Delta_D^L$  and  $V_D^R \approx 1 + \Delta_D^R$ . By eq. (2.5), we easily obtain

$$\begin{aligned} \Delta_{Dij[i \neq j]}^L &= \frac{M_{Di}^{\text{dia}} (\delta M_D^\dagger)_{ij} + \delta M_{Dij} M_{Dj}^{\text{dia}}}{|M_{Di}^{\text{dia}}|^2 - |M_{Dj}^{\text{dia}}|^2}, \\ \Delta_{Dij[i \neq j]}^R &= \frac{M_{Di}^{\text{dia}} \delta M_{Dij} + (\delta M_D^\dagger)_{ij} M_{Dj}^{\text{dia}}}{|M_{Di}^{\text{dia}}|^2 - |M_{Dj}^{\text{dia}}|^2}. \end{aligned} \quad (2.6)$$

We note that  $m_D m_D^\dagger \approx M_D^{\text{dia}} M_D^{\text{dia}\dagger}$ .

After getting the unitary matrices  $V_D^{L(R)}$ , we now discuss the charged Higgs couplings. According to eq. (2.1), the Yukawa couplings for the charged scalars are written as

$$-\mathcal{L}_Y^{H^+} = \bar{u}_L V^0 \mathbf{Y}_d D_R \phi_d^+ + \bar{u}_L V^0 \left( \epsilon_0 + \epsilon_Y V^{0\dagger} U U^\dagger V^0 \right) \mathbf{Y}_d D_R \phi_u^+. \quad (2.7)$$

In terms of eq. (2.4), the charged scalar interactions become

$$-\mathcal{L}_Y^{H^+} = \bar{u}_L V^0 \mathbf{Y}_d V_D^{R\dagger} d_R \left( \phi_d^+ - \frac{1}{\tan \beta} \phi_u^+ \right) + \frac{1}{v_d \tan \beta} \bar{u}_L V \mathbf{m}_D d_R \phi_u^+. \quad (2.8)$$

With the new physical states, the CKM matrix is modified to be  $V = V^0 V_D^{L\dagger}$ . Consequently, the first term of eq. (2.8) could be expressed by the corrected CKM matrix as  $V^0 \mathbf{Y}_d V_D^{R\dagger} = V V_D^L \mathbf{Y}_d V_D^{R\dagger}$ . Taking the leading effects of  $\epsilon_Y$ , we get

$$V_D^L \mathbf{Y}_d V_D^{R\dagger} = (1 + \Delta_D^L) \mathbf{Y}_d (1 - \Delta_D^R) \approx \mathbf{Y}_d + \Delta_D^L \mathbf{Y}_d - \mathbf{Y}_d \Delta_D^R \quad (2.9)$$

where

$$(\Delta_D^L \mathbf{Y}_d - \mathbf{Y}_d \Delta_D^R)_{ij[i \neq j]} = -\frac{\epsilon_Y \tan \beta y_t^2}{v_d (1 + \tan \beta \epsilon_3) (1 + \tan \beta \epsilon_0)} V_{i3}^{0\dagger} V_{3j}^0. \quad (2.10)$$

Since eq. (2.10) depends on the CKM matrix elements at the lowest order, by  $V = V^0 V_D^{L\dagger} \approx V^0 (1 - \Delta_D^L)$ , we obtain the relation to the corrected CKM matrix elements as

$$V_{i3}^{0\dagger} V_{33}^0 = V_{i3}^\dagger V_{33} \frac{1 + \tan \beta \epsilon_3}{1 + \tan \beta \epsilon_0}. \quad (2.11)$$

It is known that the charged Goldstone and Higgs bosons are given by [19]

$$\begin{aligned} G^+ &= \cos \beta \phi_d^+ + \sin \beta \phi_u^+ \\ H^+ &= -\sin \beta \phi_d^+ + \cos \beta \phi_u^+. \end{aligned} \quad (2.12)$$

Hence, with eqs. (2.8)–(2.11), the effective interactions for the charged Higgs coupling to  $b$ -quark and  $q$  with  $q = (c, u)$  can be written as

$$\mathcal{L}_Y^{H^+} = \left(2\sqrt{2}G_F\right)^{1/2} \tilde{V}_{qb} m_b \tan \beta \bar{q}_L b_R H^+ + h.c. \quad (2.13)$$

with

$$\tilde{V}_{qb} = V_{qb} \left[ \frac{1}{1 + \tan \beta \epsilon_3} - \frac{\epsilon_Y y_t^2}{\sin \beta \cos \beta (1 + \tan \beta \epsilon_0)^2} \right]. \quad (2.14)$$

It is easy to check that when  $\epsilon_0$  and  $\epsilon_Y$  vanish, the couplings return to the ordinary results with  $\tilde{V}_{qb} = V_{qb}$ .

In the MSSM, the one-loop corrections to  $\epsilon_0$  and  $\epsilon_Y$  are given by [13]

$$\epsilon_0 = \frac{2\alpha_s}{3\pi} \frac{\mu M_{\tilde{g}}}{M_{d_L}^2} F_2 \left( \frac{M_{\tilde{g}}^2}{M_{d_L}^2}, \frac{M_{d_R}^2}{M_{d_L}^2} \right), \quad \epsilon_Y = \frac{1}{(4\pi)^2} \frac{\mu A_u}{M_{u_L}^2} F_2 \left( \frac{M_{\tilde{g}}^2}{M_{d_L}^2}, \frac{M_{d_R}^2}{M_{d_L}^2} \right) \quad (2.15)$$

with

$$F_2(x, y) = -\frac{x \ln(x)}{(1-x)(x-y)} - \frac{y \ln(y)}{(y-1)(x-y)},$$

where  $\mu$  is the parameter describing the mixing of  $H_d$  and  $H_u$ ,  $A_U$  denotes the soft trilinear coupling and  $M_{\tilde{f}}$  with  $f = g, u_L, d_R, d_L$  represent the masses of the corresponding sfermions.

### 3. Formalisms for the decays $B^- \rightarrow \ell \bar{\nu}_\ell$ and $\bar{B} \rightarrow P(V) \ell \bar{\nu}_\ell$

In this section, we study the influence of the charged Higgs on the leptonic  $B^- \rightarrow \ell \bar{\nu}_\ell$  decays and semileptonic  $\bar{B} \rightarrow P(V) \ell \bar{\nu}_\ell$  decays, which are governed by  $b \rightarrow q \ell \bar{\nu}_\ell$  with  $q = (c, u)$  at the quark level. The effective Hamiltonian for  $b \rightarrow q \ell \bar{\nu}_\ell$  with the charged Higgs contribution is given by

$$H_{\text{eff}} = \frac{G_F V_{ub}}{\sqrt{2}} [\bar{q} \gamma_\mu (1 - \gamma_5) b \bar{\ell} \gamma^\mu (1 - \gamma_5) \nu_\ell - \delta_H \bar{q} (1 + \gamma_5) b \bar{\ell} (1 - \gamma_5) \nu_\ell] \quad (3.1)$$

with

$$\delta_H = \frac{\tilde{V}_{ub} m_b m_\ell \tan^2 \beta}{V_{ub} m_{H^+}^2}. \quad (3.2)$$

Based on the effective interaction in eq. (3.1), in the following we discuss the relevant physical quantities for various  $B$  decays.

### 3.1 Decay rate for $B^- \rightarrow \ell \bar{\nu}_\ell$

In terms of eq. (3.1), the transition amplitude for  $B^- \rightarrow \ell \bar{\nu}_\ell$  is given by

$$\begin{aligned} \langle \ell \bar{\nu}_\ell | H_{\text{eff}} | B^- \rangle &= \frac{G_F}{\sqrt{2}} V_{ub} [\langle 0 | \bar{u} \gamma_\mu (1 - \gamma_5) b | B^- \rangle \bar{\ell} \gamma^\mu (1 - \gamma_5) \nu_\ell \\ &\quad - \delta_H \langle 0 | \bar{u} (1 + \gamma_5) b | B^- \rangle \bar{\ell} (1 - \gamma_5) \nu_\ell]. \end{aligned} \quad (3.3)$$

Since the process is a leptonic decay, the QCD effect is only related to the decay constant of the  $B$  meson, which is associated the axial vector current, defined by

$$\langle 0 | \bar{u} \gamma^\mu (1 - \gamma_5) b | B^- \rangle = -i f_B p_B^\mu. \quad (3.4)$$

By equation of motion, one has

$$\langle 0 | \bar{u} \gamma_5 b | B^- \rangle \approx -i f_B \frac{m_B^2}{m_b} \quad (3.5)$$

for the pseudoscalar current. From eqs. (3.3), (3.4) and (3.5), the decay rate for  $B^- \rightarrow \ell \bar{\nu}_\ell$  with the charged Higgs contribution is expressed by

$$\frac{\Gamma^{H^+}(B^- \rightarrow \ell \bar{\nu}_\ell)}{\Gamma^{SM}(B^- \rightarrow \ell \bar{\nu}_\ell)} = \left| 1 - \delta_H \frac{m_B^2}{m_\ell m_b} \right|^2 \quad (3.6)$$

where

$$\Gamma^{SM}(B^- \rightarrow \ell \bar{\nu}_\ell) = \frac{G_F^2 |V_{ub}|^2}{8\pi} f_B^2 m_\ell^2 m_B \left( 1 - \frac{m_\ell^2}{m_B^2} \right)^2. \quad (3.7)$$

### 3.2 Differential decay rate and angular asymmetry for $\bar{B} \rightarrow P \ell \bar{\nu}_\ell$

By using the effective interaction for  $b \rightarrow q \ell \bar{\nu}_\ell$  in eq. (3.1), we write the decay amplitude for  $\bar{B} \rightarrow P \ell \bar{\nu}_\ell$  to be

$$\begin{aligned} M(\bar{B} \rightarrow P \ell \bar{\nu}_\ell) &= \langle \ell \bar{\nu}_\ell P | H_{\text{eff}} | \bar{B} \rangle = \frac{G_F V_{qb}}{\sqrt{2}} [\langle P | \bar{q} \gamma_\mu (1 - \gamma_5) b | \bar{B} \rangle \bar{\ell} \gamma^\mu (1 - \gamma_5) \nu_\ell \\ &\quad - \delta_H \langle P | \bar{q} (1 + \gamma_5) b | \bar{B} \rangle \bar{\ell} (1 - \gamma_5) \nu_\ell]. \end{aligned} \quad (3.8)$$

To get the hadronic QCD effect, we parametrize the  $\bar{B} \rightarrow P$  transition as

$$\langle P(p_P) | \bar{q} \gamma^\mu b | \bar{B}(p_B) \rangle = f_+^P(q^2) \left( P^\mu - \frac{P \cdot q}{q^2} q^\mu \right) + f_0^P(q^2) \frac{P \cdot q}{q^2} q^\mu, \quad (3.9)$$

$$\langle P(p_P) | \bar{q} b | \bar{B}(p_B) \rangle \approx f_0^P(q^2) \frac{P \cdot q}{m_b}, \quad (3.10)$$

with  $P = p_B + p_P$  and  $q = p_B - p_P$ . To calculate the decay rate, we choose the coordinates for various particles as follows:

$$\begin{aligned} q^2 &= (\sqrt{q^2}, 0, 0, 0), & p_B &= (E_B, 0, 0, |\vec{p}_P|), \\ p_P &= (E_P, 0, 0, |\vec{p}_P|), & p_\ell &= (E_\ell, |\vec{p}_\ell| \sin \theta, 0, |\vec{p}_\ell| \cos \theta), \end{aligned} \quad (3.11)$$

where  $E_P = (m_B^2 - q^2 - m_P^2)/(2\sqrt{q^2})$ ,  $|\vec{p}_P| = \sqrt{E_P^2 - m_P^2}$ ,  $E_\ell = (q^2 + m_\ell^2)/(2\sqrt{q^2})$  and  $|\vec{p}_\ell| = (q^2 - m_\ell^2)/(2\sqrt{q^2})$ . It is clear that  $\theta$  is defined as the polar angle of the lepton momentum relative to the moving direction of the  $B$ -meson in the  $q^2$  rest frame. The differential decay rate for  $\bar{B} \rightarrow P \ell \bar{\nu}_\ell$  as a function of  $q^2$  and  $\theta$  is given by

$$\begin{aligned} \frac{d\Gamma_P}{dq^2 d\cos\theta} &= \frac{G_F^2 |V_{ub}|^2 m_B^3}{2^8 \pi^3} \sqrt{(1-s+\hat{m}_P^2)^2 - 4\hat{m}_P^2} \left(1 - \frac{\hat{m}_\ell^2}{s}\right)^2 \\ &\times [\Gamma_1^P + \Gamma_2^P \cos\theta + \Gamma_3^P \cos^2\theta], \end{aligned} \quad (3.12)$$

$$\begin{aligned} \Gamma_1^P &= f_+^{P2}(q^2) \hat{P}_P^2 + \hat{m}_\ell^2 s \left| \frac{1-s-\hat{m}_P^2}{s} f_+^P(q^2) + C_2 \right|^2, \\ \Gamma_2^P &= 2\hat{m}_\ell^2 \hat{P}_P^2 \left[ f_+^P(q^2) C_2 - \frac{1-s-\hat{m}_P^2}{s} f_+^{P2}(q^2) \right], \\ \Gamma_3^P &= -f_+^{P2}(q^2) \hat{P}_P^2 \left(1 - \frac{\hat{m}_\ell^2}{s}\right), \end{aligned} \quad (3.13)$$

where  $s = q^2/m_B^2$ ,  $\hat{m}_i = m_i/m_B$  and

$$\begin{aligned} \hat{P}_P &= 2\sqrt{s} |\vec{p}_P|/m_B = \sqrt{(1-s-\hat{m}_P^2)^2 - 4s\hat{m}_P^2}, \\ C_2 &= f_+^P(q^2) + (f_0^P(q^2) - f_+^P(q^2)) \frac{1-\hat{m}_P^2}{s} - \delta_H \frac{1-\hat{m}_P^2}{\hat{m}_\ell \hat{m}_b} f_0^P(q^2) \end{aligned} \quad (3.14)$$

Since the differential decay rate in eq. (3.12) involves the polar angle of the lepton, we can define an angular asymmetry to be

$$\mathcal{A}(q^2) = \frac{\int_0^{\pi/2} d\cos\theta d\Gamma/(dq^2 d\cos\theta) - \int_{\pi/2}^\pi d\cos\theta d\Gamma/(dq^2 d\cos\theta)}{\int_0^{\pi/2} d\cos\theta d\Gamma/(dq^2 d\cos\theta) + \int_{\pi/2}^\pi d\cos\theta d\Gamma/(dq^2 d\cos\theta)}. \quad (3.15)$$

Explicitly, for  $\bar{B} \rightarrow P \ell \bar{\nu}_\ell$ , the asymmetry is given by

$$\mathcal{A}_P(s) = -\frac{\Gamma_2^P}{2\Gamma_1^P + 2/3\Gamma_3^P}. \quad (3.16)$$

### 3.3 Differential decay rate and angular asymmetry for $\bar{B} \rightarrow V \ell \bar{\nu}_\ell$

Similar to eq. (3.8), for  $\bar{B} \rightarrow V \ell \bar{\nu}_\ell$ , we need to know the form factors in the  $B \rightarrow V$  transition. As usual, we parametrize the transition form factors to be

$$\langle V(p_V, \varepsilon) | \bar{q} \gamma_\mu b | \bar{B}(p_B) \rangle = i \frac{V^V(q^2)}{m_B + m_V} \epsilon_{\mu\alpha\beta\rho} \varepsilon^{*\alpha} P^\beta q^\rho,$$

$$\begin{aligned}
 \langle V(p_V, \varepsilon) | \bar{q} \gamma^\mu \gamma_5 b | \bar{B}(p_1) \rangle &= 2m_V A_0^V(q^2) \frac{\varepsilon^* \cdot q}{q^2} q_\mu + (m_B + m_V) A_1^V(q^2) \left( \varepsilon_\mu^* - \frac{\varepsilon^* \cdot q}{q^2} q_\mu \right) \\
 &\quad - A_2^V(q^2) \frac{\varepsilon^* \cdot q}{m_B + m_V} \left( P_\mu - \frac{P \cdot q}{q^2} q_\mu \right). \tag{3.17}
 \end{aligned}$$

By equation of motion, we have

$$\langle V(p_V, \varepsilon) | \bar{q} \gamma_5 b | \bar{B}(p_B) \rangle = -\frac{2m_V}{m_b} \varepsilon^* \cdot q A_0^V(q^2). \tag{3.18}$$

Consequently, the decay amplitude is expressed by

$$\begin{aligned}
 M(\bar{B} \rightarrow V \ell \bar{\nu}_\ell) &= \frac{G_F V_{ub}}{\sqrt{2}} \left[ T_\mu \bar{\ell} \gamma^\mu (1 - \gamma_5) \nu_\ell + 2 \frac{\varepsilon^* \cdot q}{m_B} L_1 \bar{\ell} \not{p}_V (1 - \gamma_5) \nu_\ell \right. \\
 &\quad \left. + m_\ell \frac{\varepsilon^* \cdot q}{m_B} L_2 \bar{\ell} (1 - \gamma_5) \nu_\ell \right] \tag{3.19}
 \end{aligned}$$

where

$$\begin{aligned}
 T_\mu &= i \frac{2V^V(q^2)}{m_B + m_V} \varepsilon_{\mu\alpha\beta\rho} \varepsilon^{*\alpha} p_K^\beta q^\rho - (m_B + m_V) A_1^V(q^2) \left( \varepsilon_\mu^* - \frac{\varepsilon^* \cdot q}{q^2} q_\mu \right), \\
 L_1 &= \frac{A_2^V(q^2)}{1 + \hat{m}_V}, \quad L_2 = \frac{1 - (1 - \hat{m}_V^2)/s}{1 + \hat{m}_V} A_2^V(q^2) - 2\hat{m}_V \left( \frac{1}{s} - \frac{\delta_H}{\hat{m}_\ell \hat{m}_b} \right) A_0^V(q^2). \tag{3.20}
 \end{aligned}$$

The differential decay rate for  $\bar{B} \rightarrow V \ell \bar{\nu}_\ell$  as a function of  $q^2$  and  $\theta$  is given by

$$\begin{aligned}
 \frac{d\Gamma_V}{dq^2 d\cos\theta} &= \frac{G_F^2 |V_{ub}|^2 m_B^3}{2^8 \pi^3} \sqrt{(1 - s + \hat{m}_V^2)^2 - 4\hat{m}_V^2} \left( 1 - \frac{\hat{m}_\ell^2}{s} \right)^2 \\
 &\quad \times [\Gamma_1^V + \Gamma_2^V \cos\theta + \Gamma_3^V \cos^2\theta + \Gamma_4^V \sin^2\theta] \tag{3.21}
 \end{aligned}$$

where

$$\begin{aligned}
 \Gamma_1^V &= s \left[ 2 \left( \frac{V^V(q^2)}{1 + \hat{m}_V} \right)^2 \hat{P}_V^2 + \left( 3 + \frac{\hat{P}_V^2}{4s\hat{m}_V^2} \right) (1 + \hat{m}_V)^2 A_1^{V2}(q^2) \right] \\
 &\quad + s L_1^2 \left[ \frac{E_V^2}{m_V^2} - 1 \right] \left[ \hat{P}_V^2 \left( 1 + \frac{\hat{m}_\ell^2}{s} \right) + 4\hat{m}_\ell^2 \hat{m}_V^2 \right] \\
 &\quad - 2s(1 - s - \hat{m}_V^2) \left[ \frac{E_V^2}{m_V^2} - 1 \right] (1 + \hat{m}_V) A_1^V(q^2) L_1 \\
 &\quad + \hat{m}_\ell^2 s^2 \left[ \frac{E_V^2}{m_V^2} - 1 \right] L_2^2 + 2\hat{m}_\ell^2 s(1 - s - \hat{m}_V^2) \left[ \frac{E_V^2}{m_V^2} - 1 \right] L_1 L_2,
 \end{aligned}$$

$$\begin{aligned}
 \Gamma_2^V &= 16\hat{P}_V V^V(q^2) A_1^V(q^2) + 2\hat{m}_\ell^2 (1 - s - \hat{m}_V^2) \hat{P}_V \left[ \frac{E_V^2}{m_V^2} - 1 \right] L_1^2 \\
 &\quad + 2\hat{m}_\ell^2 s \hat{P}_V \left[ \frac{E_V^2}{m_V^2} - 1 \right] L_1 L_2 - \frac{\hat{m}_\ell^2}{2s\hat{m}_V^2} (1 - s - \hat{m}_V^2)^2 \hat{P}_V (1 + \hat{m}_V) A_1^V(q^2) L_1 \\
 &\quad - \frac{\hat{m}_\ell^2}{2\hat{m}_V^2} (1 - s - \hat{m}_V^2) \hat{P}_V (1 + \hat{m}_V) A_1^V(q^2) L_2,
 \end{aligned}$$



$$\begin{aligned}\Gamma_3^V &= -s \left(1 - \frac{\hat{m}_\ell^2}{s}\right) \left[ \frac{E_V^2}{M_V^2} (1 + \hat{m}_V)^2 A_1^{V2}(q^2) + \hat{P}_V^2 \left( \frac{E_V^2}{m_V^2} - 1 \right) L_1^2 \right] \\ &\quad + \frac{\hat{P}_V^2}{2\hat{m}_V^2} \left(1 - \frac{\hat{m}_\ell^2}{s}\right) (1 - s - \hat{m}_V^2)(1 + \hat{m}_V) A_1(q^2) L_2 \\ \Gamma_4^V &= -s \left(1 - \frac{\hat{m}_\ell^2}{s}\right) \left[ \hat{P}_V^2 \left( \frac{V^V(q^2)}{1 + \hat{m}_V} \right)^2 + (1 + \hat{m}_V)^2 A_1^{V2}(q^2) \right]\end{aligned}$$

with  $\hat{P}_V = 2\sqrt{s}|\vec{p}_V|/m_B = \sqrt{(1 - s - \hat{m}_V^2)^2 - 4s\hat{m}_V^2}$ . In addition, from eqs. (3.15) and (3.21), we obtain the angular asymmetry for  $\bar{B} \rightarrow V\ell\bar{\nu}_\ell$  to be

$$\mathcal{A}_V(s) = -\frac{\Gamma_2^V}{2\Gamma_1^V + 2/3(\Gamma_3^V + 2\Gamma_4^V)}. \quad (3.22)$$

#### 4. Numerical analysis

In the numerical calculations, the model-independence inputs are used as follows:  $G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$ ,  $m_b = 4.4 \text{ GeV}$ , and  $m_B = 5.28 \text{ GeV}$ . In addition, to reduce the unknown parameters in eq. (2.15) for the MSSM, we set  $|\mu| \approx |A_U| \equiv \bar{\mu}$  and  $M_{\tilde{d}_L} \approx M_{\tilde{d}_R} \approx M_{\tilde{u}_L} \approx M_{\tilde{g}} = M_S$  so that the loop integral is simplified to be a constant with  $F(x, y) = 1/2$ . Subsequently, the one-loop corrected effects are simplified as

$$\epsilon_0 \approx \pm \frac{\alpha_s}{3\pi} \frac{\bar{\mu}}{M_S}, \quad \epsilon_Y \approx \pm \frac{1}{2(4\pi)^2} \frac{\bar{\mu}^2}{M_S^2}, \quad (4.1)$$

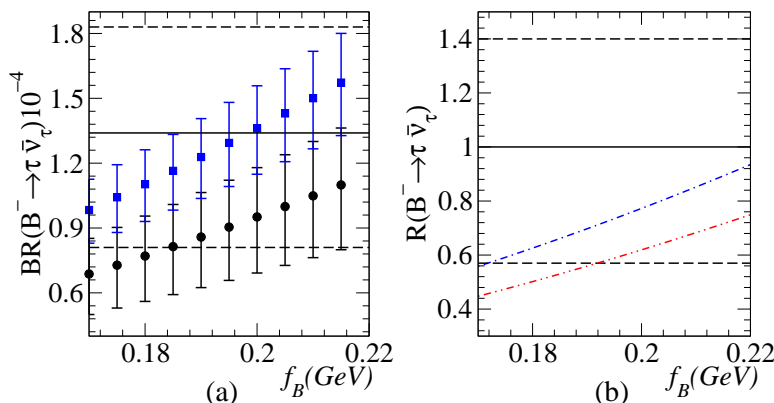
where the signs depend on  $\mu$  and  $A_U$ , respectively. Hence, we have four possibilities for the sign combinations in  $\epsilon_0$  and  $\epsilon_Y$ . Clearly, based on the assumption, besides  $\tan\beta$  and the charged Higgs mass, now only one new parameter, denoted by  $X = \bar{\mu}/M_S$ , is introduced in the charged Higgs couplings. To study the charged Higgs effects at a large  $\tan\beta$  region, we fix  $\tan\beta = 50$ .

##### 4.1 $B^- \rightarrow \tau\nu_\tau$

According to eq. (3.7), it is clear that the BR for  $B^- \rightarrow \tau\nu_\tau$  in the SM depends on two main parameters  $f_B$  and  $V_{ub}$ . To see their contributions, we calculate the BR with different values of  $f_B$ . For each value of  $f_B$ , we consider two sets of  $V_{ub}$ , *i.e.*,  $(4.39 \pm 0.33) \times 10^{-3}$  [1] and  $(3.67 \pm 0.47) \times 10^{-3}$  [5], extracted from the inclusive and exclusive semileptonic  $B$  decays, respectively.

We present the results in figure 1(a) where the squares (circles) in the central values denote those calculated with the bigger (smaller) value of  $V_{ub}$  and the solid line displays the central value of the data, while the dashed lines are the upper and lower values with  $1\sigma$  errors, respectively. From the figure, we notice that with the smaller  $V_{ub}$ , the value of  $f_B = 0.216 \pm 0.022 \text{ GeV}$  given by the unquenched lattice is still favorable [20]. To reduce the uncertainty from the CKM matrix element, we propose a quantity, defined by the ratio

$$R(B^- \rightarrow \tau\nu_\tau) = \frac{BR(B^- \rightarrow \tau\nu_\tau)}{BR(\bar{B} \rightarrow \pi^+ e^- \bar{\nu}_e)}. \quad (4.2)$$



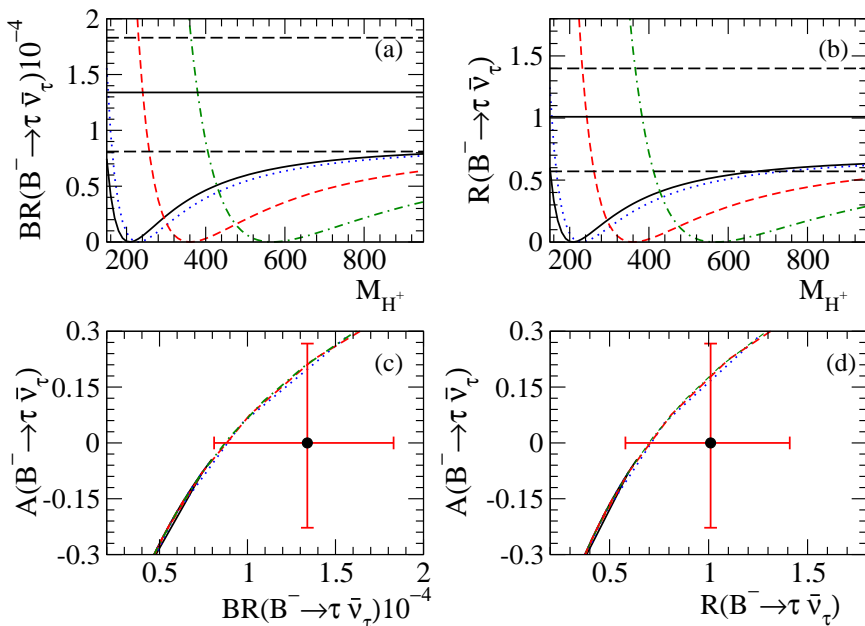
**Figure 1:** (a) BR (in units of  $10^{-4}$ ) and (b)  $R$  calculated by LFQM (dot-dashed) and LCSRs (dot-dot-dashed) for the decay of  $B^- \rightarrow \tau \bar{\nu}_\tau$  with respect to  $f_B$ , where the squares and circles stand for  $|V_{ub}| = (4.39 \pm 0.33) \times 10^{-3}$  and  $(3.67 \pm 0.47) \times 10^{-3}$  and the solid and dashed lines represent the central value and the  $1\sigma$  errors of the data, respectively.

It is clear that the ratio of  $R(B^- \rightarrow \tau \bar{\nu}_\tau)$  in eq. (4.2) could directly reflect the charged Higgs effect in  $B^- \rightarrow \tau \bar{\nu}_\tau$  as the charged contribution to  $\bar{B} \rightarrow \pi^+ e^- \bar{\nu}_e$  is suppressed. However, we introduce new theoretical uncertainty arising from the transition form factor  $f_+^\pi(q^2)$  defined by eq. (3.9). To see the influence of uncertainty on the ratio  $R(B^- \rightarrow \tau \bar{\nu}_\tau)$ , we use two different QCD approaches of the light-front quark model (LFQM) [21] and light cone sum rules (LCSRs) [22] to estimate the form factor. With  $|V_{ub}| = 3.67 \times 10^{-3}$ , we get that the former predicts  $BR(\bar{B} \rightarrow \pi^+ e^- \bar{\nu}_e) = 1.25 \times 10^{-4}$  while the latter  $BR(\bar{B} \rightarrow \pi^+ e^- \bar{\nu}_e) = 1.55 \times 10^{-4}$ , which are consistent with the data of  $(1.33 \pm 0.22) \times 10^{-4}$  [5]. From the results, we see that the error from the uncertainty of  $f_+^\pi(q^2)$  on the  $R(B^- \rightarrow \tau \bar{\nu}_\tau)$  could be around 20% which is still less than the error of 40% from  $V_{ub}$ . To be more clear, in figure 1(b) we display the ratio  $R(B^- \rightarrow \tau \bar{\nu}_\tau)$  by LFQM (dot-dashed) and LCSRs (dot-dot-dashed) in the SM, where the solid and dashed lines denote the central value and errors of the current data  $R(B^- \rightarrow \tau \bar{\nu}_\tau) = 1.02 \pm 0.40$ , respectively.

In terms of eq. (4.1), we now study the influence of the charged Higgs. First of all, to understand how the charged Higgs affects  $B^- \rightarrow \tau \bar{\nu}_\tau$  directly, we display  $BR(B^- \rightarrow \tau \bar{\nu}_\tau)$  as a function of the charged Higgs mass in figure 2(a), where we have taken  $X = 1$ ,  $f_B = 0.19$  GeV and  $|V_{ub}| = 3.67 \times 10^{-3}$ . Since there is a two-fold ambiguity in sign for each  $(e_0, e_Y)$ , the solid, dotted, dashed and dot-dashed lines correspond to the possible sign combinations denoted by  $(+, +)$ ,  $(+, -)$ ,  $(-, +)$  and  $(-, -)$ , respectively. From the figure, we see that the decay  $B^- \rightarrow \tau \bar{\nu}_\tau$  could exclude some parameter space. To remove the uncertainty of  $V_{ub}$ , in figure 2(b) we show the effects of the charged Higgs on  $R(B^- \rightarrow \tau \bar{\nu}_\tau)$ . In order to make the new physics effects more clearly, we define another physical quantity as

$$\mathcal{A}(B^- \rightarrow \tau \bar{\nu}_\tau) = \frac{R(B^- \rightarrow \tau \bar{\nu}_\tau) - R^{SM}(B^- \rightarrow \tau \bar{\nu}_\tau)}{R(B^- \rightarrow \tau \bar{\nu}_\tau) + R^{SM}(B^- \rightarrow \tau \bar{\nu}_\tau)}. \quad (4.3)$$

Although the quantity  $R(B^- \rightarrow \tau \bar{\nu}_\tau)$  still depends on  $f_B$  and  $f_+^\pi(q^2)$ , the new quantity



**Figure 2:** (a) BR (in units of  $10^{-4}$ ) and (b)  $R$  for  $B^- \rightarrow \tau \bar{\nu}_\tau$  as a function of  $M_{H^+}$  and (c)[(d)]  $\mathcal{A}(B^- \rightarrow \tau \bar{\nu}_\tau)$  with respect to  $BR(B^- \rightarrow \tau \bar{\nu}_\tau)$  [ $R(B^- \rightarrow \tau \bar{\nu}_\tau)$ ], where the solid, dotted, dashed and dot-dashed lines correspond to the possible sign combinations of  $(e_0, e_Y)$  denoted by  $(+, +)$ ,  $(+, -)$ ,  $(-, +)$  and  $(-, -)$ , respectively, and the data with errors are included.

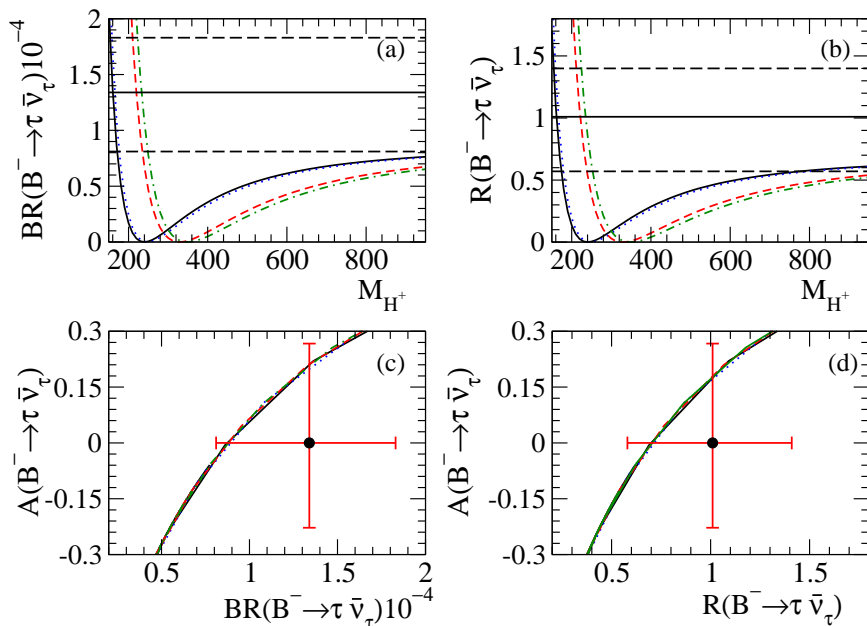
$\mathcal{A}(B^- \rightarrow \tau \bar{\nu}_\tau)$  reduces their dependences. That is, if a nonzero value of  $\mathcal{A}(B^- \rightarrow \tau \bar{\nu}_\tau)$  is measured, it shows the existence of new physics definitely. We present the charged Higgs contributions to  $\mathcal{A}(B^- \rightarrow \tau \bar{\nu}_\tau)$  with respect to  $BR(B^- \rightarrow \tau \bar{\nu}_\tau)$  and  $R(B^- \rightarrow \tau \bar{\nu}_\tau)$  in figure 2(c) and (d), respectively, where we also display the current bounds. Clearly,  $\mathcal{A}(B^- \rightarrow \tau \bar{\nu}_\tau) \sim 10\%$  is easy to reach by the charged Higgs effects in the MSSM. We note that the new physical quantity  $\mathcal{A}(B^- \rightarrow \tau \bar{\nu}_\tau)$  is not sensitive to the signs in  $e_0$  and  $e_Y$ . Similarly, we show the results with  $X = 0.5$  in figure 3.

#### 4.2 $\bar{B} \rightarrow (\pi^+, D^+) \ell \bar{\nu}_\ell$

Besides the CKM matrix element, the main theoretical uncertainty for  $\bar{B} \rightarrow P \ell \bar{\nu}_\ell$  is from the  $B \rightarrow P$  transition form factors. For numerical estimations, we employ the results of the LFQM [21] in which the form factors as a function of  $q^2$  are parametrized by

$$f^P(q^2) = \frac{f^P(0)}{1 - aq^2/m_B^2 + b(q^2/m_B^2)^2}, \quad (4.4)$$

and the fitting values of parameters  $a$  and  $b$  are shown in table 1. To check the contributions of the input form factors in the SM, we present BRs for  $\bar{B} \rightarrow (\pi^+, D^+) \ell \bar{\nu}_\ell$  in table 2, where we have used  $|V_{ub}| = 3.67 \times 10^{-3}$  and  $|V_{cb}| = (41.3 \pm 0.15) \times 10^{-3}$  [5]. It is clear that for the light lepton production, the results are consistent with the data. Since the new coupling of the charged Higgs is associated with the lepton mass, it is easily to understand that the



**Figure 3:** Legend is the same as figure 2 but  $X = 0.5$ .

$f^P(q^2)$	$f^P(0)$	$a$	$b$	$f^P(q^2)$	$f^P(0)$	$a$	$b$
$f_+^\pi(q^2)$	0.25	1.73	0.95	$f_0^\pi(q^2)$	0.25	0.84	0.10
$f_+^D(q^2)$	0.67	1.25	0.39	$f_0^D(q^2)$	0.67	0.65	0.00

**Table 1:** The transition form factors for  $B \rightarrow (\pi, D)$  calculated by the LFQM [21].

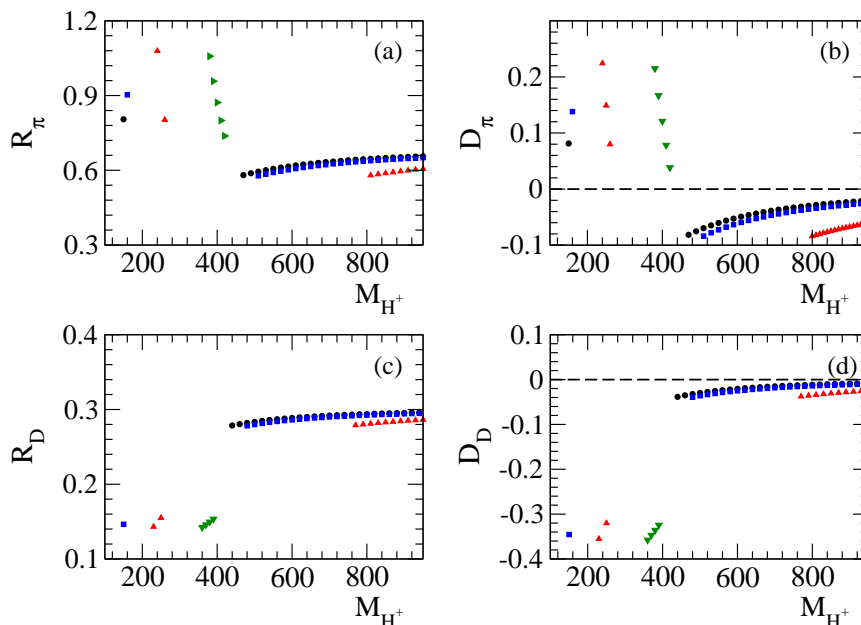
Mode	$\bar{B} \rightarrow \pi^+ \ell^- \bar{\nu}_\ell$	$\bar{B} \rightarrow \pi^+ \tau^- \bar{\nu}_\tau$	$\bar{B} \rightarrow D^+ \ell^- \bar{\nu}_\ell$	$\bar{B} \rightarrow D^+ \tau^- \bar{\nu}_\tau$
SM	$(1.25 \pm 0.23)10^{-4}$	$(0.85 \pm 0.15)10^{-4}$	$(2.29 \pm 0.12)\%$	$(0.69 \pm 0.04)\%$
Experiment [5]	$(1.33 \pm 0.22)10^{-4}$		$(2.12 \pm 0.20)\%$	

**Table 2:** BRs for  $\bar{B} \rightarrow \pi^+ \ell^- \bar{\nu}_\ell$  with  $|V_{ub}| = (3.67 \pm 0.47) \times 10^{-3}$  and  $\bar{B} \rightarrow D^+ \ell^- \bar{\nu}_\ell$  with  $|V_{cb}| = (41.3 \pm 1.5) \times 10^{-3}$  in the SM.

effects of the charged Higgs will not significantly affect the light leptonic decays. Hence, in our analysis, we will only concentrate on the  $\tau$  decay modes.

Since  $\bar{B} \rightarrow (\pi^+, D^+) \tau \bar{\nu}_\tau$  have not been observed yet, we take  $R(B^- \rightarrow \tau \bar{\nu}_\tau) = 1.02 \pm 0.40$  as a constraint. To reduce the theoretical uncertainty from the CKM matrix elements, we consider the ratio

$$R_P = \frac{BR(\bar{B} \rightarrow P \tau \bar{\nu}_\tau)}{BR(\bar{B} \rightarrow P \ell \bar{\nu}_\ell)} \quad (4.5)$$



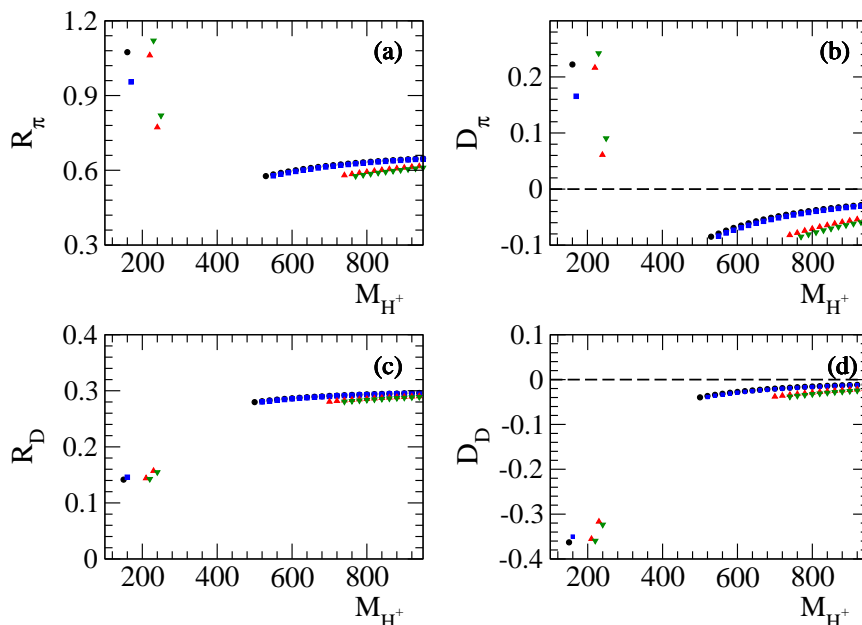
**Figure 4:** (a)[(c)] denotes the  $R_\pi$ [ $R_D$ ] and (b)[(d)] displays  $D_\pi$ [ $D_D$ ] with respect to  $M_{H^+}$  for  $X = 1$ , where the circle, square, triangle-up and triangle down represent the sign combinations of  $(e_0, e_Y)$  such as  $(+, +)$ ,  $(+, -)$ ,  $(-, +)$  and  $(-, -)$ .

instead of  $BR(\bar{B} \rightarrow P\tau\bar{\nu}_\tau)$ . In addition, to illustrate new physics clearly, we also define

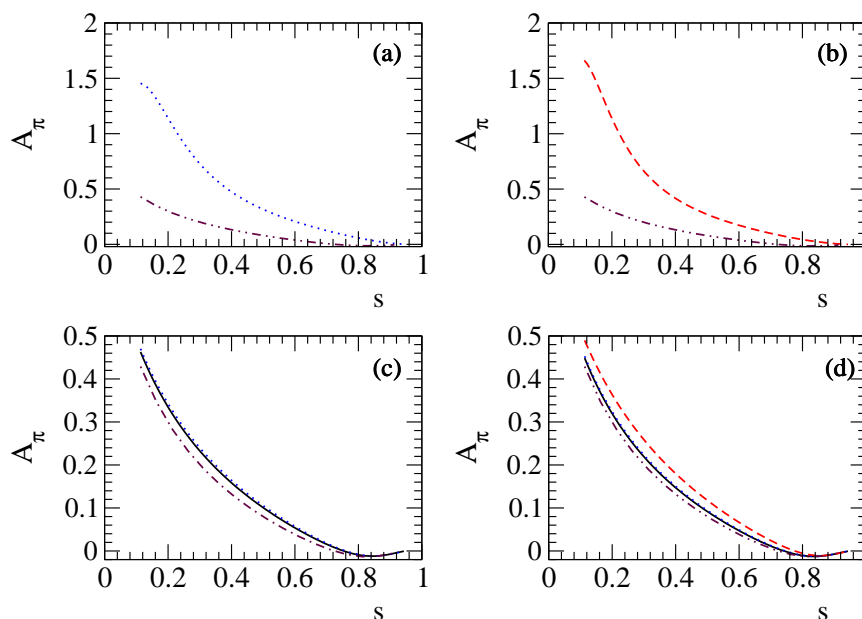
$$\mathcal{D}_P = \frac{R_P - R_P^{SM}}{R_P + R_P^{SM}}. \quad (4.6)$$

If a non-zero value of  $\mathcal{D}_P$  is observed, it must indicate the existence of new physics. Hence, according to eq. (3.12),  $R_P$  and  $\mathcal{D}_P$  for  $P = (\pi^+, D^+)$  with  $X = 1$  are displayed in figure 4, where the circle, square, triangle-up and triangle-down symbols correspond to the possible signs for  $e_0$  and  $e_Y$ , expressed by  $(+, +)$ ,  $(+, -)$ ,  $(-, +)$  and  $(-, -)$ , respectively. Similar analysis with  $X = 0.5$  is presented in figure 5. By the figures, we see that the input  $R(B^- \rightarrow \tau\bar{\nu}_\tau)$  has given a strict constraint on the signs of  $e_0$  and  $e_Y$  and the parameters of  $X = |\mu|/M_S$  and  $M_{H^+}$ . Even so, we still can have  $O(10\%)$  deviation in  $\mathcal{D}_P$  when  $M_{H^+}$  is less than 400 GeV.

Note that apart from the BR related quantities, the angular distribution asymmetry defined in eq. (3.16) could also be used to examine the effects beyond the SM [23]. We display the contributions of the charged Higgs with  $X = 1$  to  $\mathcal{A}_P$  in figure 6 (7) for  $\bar{B} \rightarrow \pi^+\tau\bar{\nu}_\tau$  ( $\bar{B} \rightarrow D^+\tau\bar{\nu}_\tau$ ). In the figures, (a), (b), (c) and (d) correspond to  $M_{H^+} = 160, 230, 650$  and  $850$  GeV, the solid, dotted, dashed and dot-dashed denote the sign combinations of  $(e_0, e_Y) = (+, +)$ ,  $(+, -)$ ,  $(-, +)$  and  $(-, -)$ , and the dash-dotted-dotted line stands for the SM result, respectively. From the figures, it is clear that the charged Higgs contributions in the light  $M_{H^+}$  region could significantly affect the angular asymmetries. We note that due to the constraint  $R(B^- \rightarrow \tau\bar{\nu}_\tau) = 1.02 \pm 0.40$ , some sign combinations have been excluded with the same  $M_{H^+}$ .



**Figure 5:** Legend is the same as figure 4 but  $X = 0.5$ .

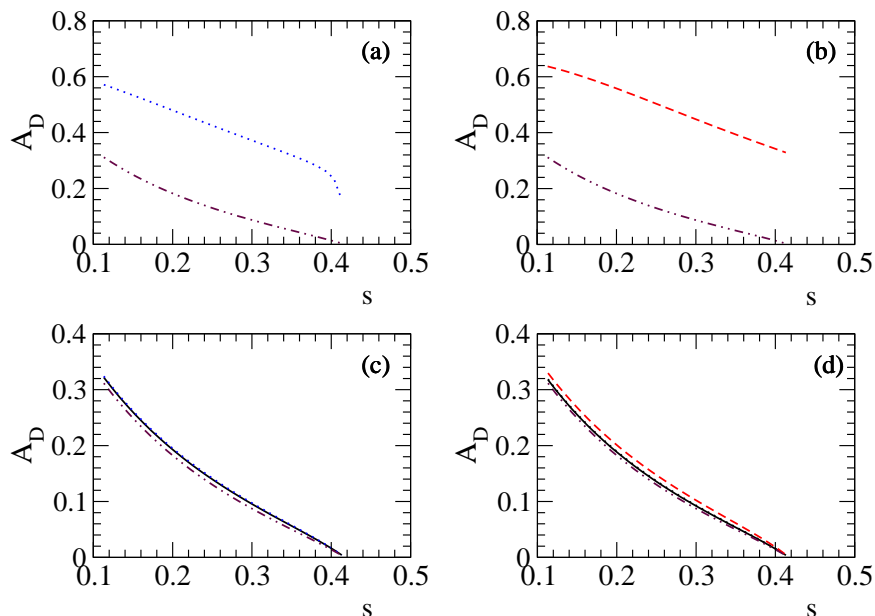


**Figure 6:** Angular asymmetries for  $\bar{B} \rightarrow \pi^+ \tau \bar{\nu}_\tau$  with  $M_{H^+} =$  (a) 160 GeV, (b) 230 GeV, (c) 650 GeV and (d) 850 GeV, where the solid, dotted, dashed, dash-dotted lines correspond to the sign combinations of  $(e_0, e_Y)$ , expressed by  $(+, +)$ ,  $(+, -)$ ,  $(-, +)$ ,  $(-, -)$ , respectively, and the dash-dotted-dotted lines represent the SM results.

### 4.3 $\bar{B} \rightarrow (\rho^+, D^{*+}) \ell \bar{\nu}_\ell$

The form factors in  $B \rightarrow (\rho, D^*)$  are parametrized by

$$f^V(q^2) = \frac{f^V(0)}{1 - aq^2/m_B^2 + b(q^2/m_B^2)^2} \quad (4.7)$$



**Figure 7:** Legend is the same as figure 6 but for  $\bar{B} \rightarrow D^+ \tau \bar{\nu}_\tau$ .

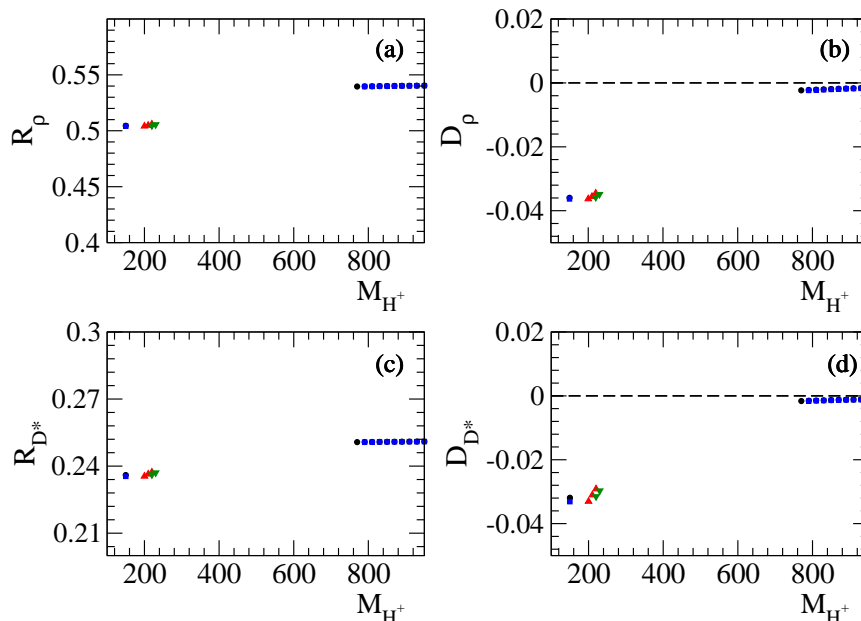
$f^V(q^2)$	$f^V(0)$	$a$	$b$	$f^V(q^2)$	$f^V(0)$	$a$	$b$
$V^\rho(q^2)$	0.27	1.84	1.28	$A_0^\rho(q^2)$	0.28	1.73	1.20
$A_1^\rho(q^2)$	0.22	0.95	0.21	$A_2^\rho(q^2)$	0.20	1.65	1.05
$V^{D^*}(q^2)$	0.75	1.29	0.45	$A_0^{D^*}(q^2)$	0.64	1.30	0.31
$A_1^{D^*}(q^2)$	0.63	0.65	0.02	$A_2^{D^*}(q^2)$	0.61	1.14	0.52

**Table 3:** The transition form factors for  $B \rightarrow (\rho, D^*)$  calculated by the LFQM [21].

Mode	$\bar{B} \rightarrow \rho^+ \ell^- \nu_\ell$	$\bar{B} \rightarrow \rho^+ \tau^- \nu_\tau$	$\bar{B} \rightarrow D^{*+} \ell^- \nu_\ell$	$\bar{B} \rightarrow D^{*+} \tau^- \nu_\tau$
SM	$(3.18 \pm 0.56)10^{-4}$	$(1.73 \pm 0.31)10^{-4}$	$(5.60 \pm 0.29)\%$	$(1.41 \pm 0.07)\%$
Exp [5]	$(2.6 \pm 0.7)10^{-4}$		$(5.34 \pm 0.20)\%$	

**Table 4:** BRs for  $\bar{B} \rightarrow \rho^+ \ell^- \bar{\nu}_\ell$  with  $|V_{ub}| = (3.67 \pm 0.47) \times 10^{-3}$  and  $\bar{B} \rightarrow D^{*+} \ell^- \bar{\nu}_\ell$  with  $|V_{cb}| = (41.3 \pm 1.5) \times 10^{-3}$  in the SM.

with  $a$  and  $b$  given in table 3. Based on these form factors and eq. (3.21), the BRs in the SM are shown in table 4. It is clear that for the light lepton production, the BRs are consistent with the current experimental data. By using the same form factors to the processes associated with the  $\tau$  production, if any significant deviation from the predictions of the SM is found, it should indicate new physics.



**Figure 8:** (a)[(c)] denotes  $R_\rho[R_{D^*}]$  and (b)[(d)] displays  $D_\rho[D_{D^*}]$  as functions of  $M_{H^+}$  for  $X = 0.5$  with  $R(B^- \rightarrow \tau\bar{\nu}_\tau) = 1.02 \pm 0.40$ . Legend is the same as figure 4.

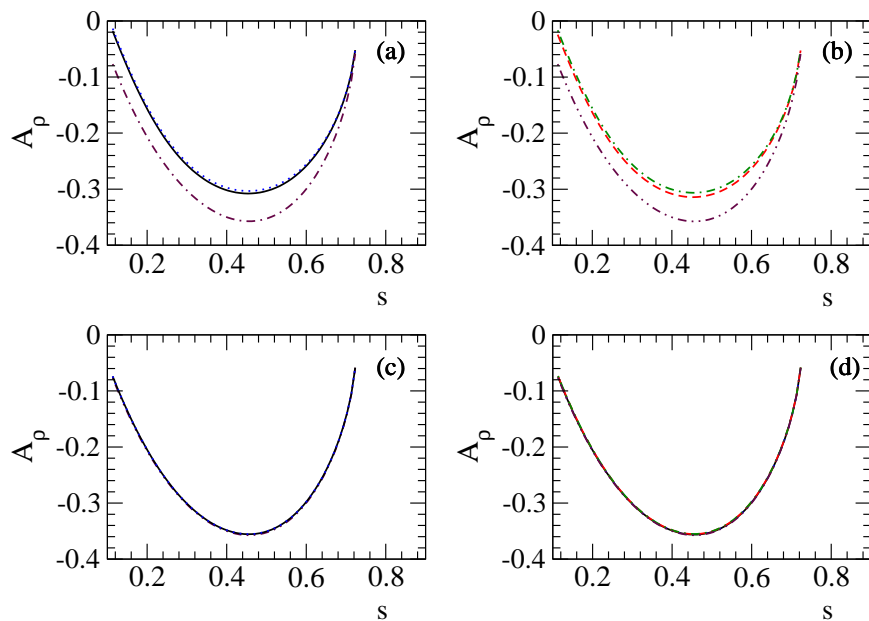
Similar to the decays  $\bar{B} \rightarrow (\pi^+, D^+)\ell\bar{\nu}_\ell$ , we define

$$R_V = \frac{BR(\bar{B} \rightarrow V\tau\bar{\nu}_\tau)}{BR(\bar{B} \rightarrow V\ell\bar{\nu}_\ell)} \quad \text{and} \quad \mathcal{D}_V = \frac{R_V - R_V^{SM}}{R_V + R_V^{SM}}. \quad (4.8)$$

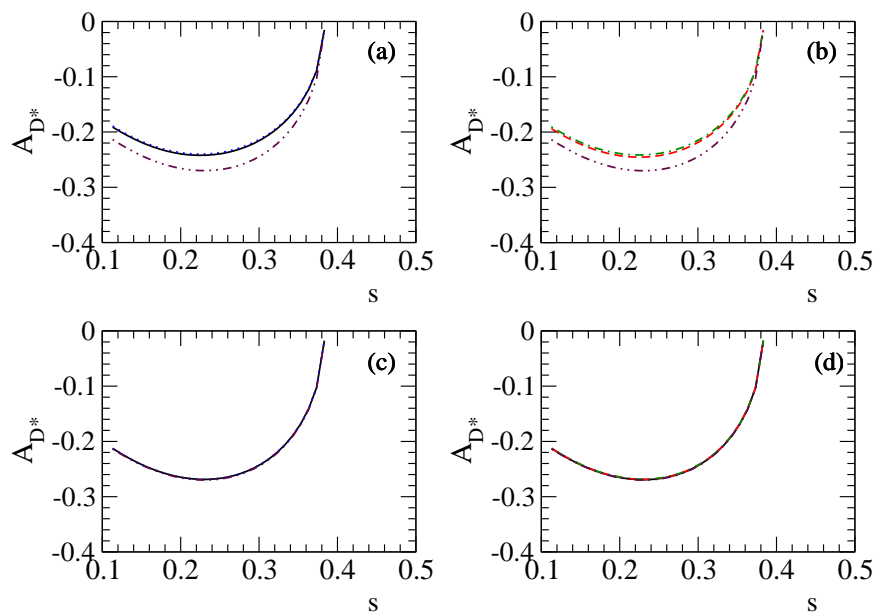
The contributions of the charged Higgs are presented in figure 8 with  $X = 0.5$ . To constrain the free parameters, we have taken  $R(B^- \rightarrow \tau\bar{\nu}_\tau) = 1.02 \pm 0.40$ . From the results, we see that the charged Higgs contributions to  $\mathcal{D}_V$  are only at few percent. In addition, we also display the angular asymmetries for  $\bar{B} \rightarrow (\rho^+, D^{+*})\tau\bar{\nu}_\tau$  in figures 9 and 10. We find that the influence of the light charged Higgs on  $\mathcal{A}_\rho$  is larger than that on  $\mathcal{A}_{D^*}$ . However, the contributions from the heavy charged Higgs are the same as the predictions in the SM.

Finally, we make some comparisons in  $\bar{B} \rightarrow P\tau\bar{\nu}_\tau$  and  $\bar{B} \rightarrow V\tau\bar{\nu}_\tau$ . For  $\bar{B} \rightarrow P\ell\bar{\nu}_\ell$ , according to eq. (3.12), one finds that the dominant effects for the BRs, which do not vanish in the limit of  $m_\ell = 0$ , are  $\propto f^{P2}(q^2)\hat{P}_P^2$ . Although the terms directly related to the lepton mass in the form of  $\hat{m}_\ell^2 f_+^{P2}(q^2)$ , for the  $\tau$  modes, the mass effects could have  $O(10\%)$  in order of magnitude. Since the new charged Higgs contributions appear in the terms associated with  $\hat{m}_\ell^2 f_0^{P2}(q^2)$ , it is expected that in average the influence of the charged Higgs could be as large as  $O(10\%)$ , which is consistent with the results shown in the figure 4. Furthermore, since the lepton angular asymmetry is associated with  $\hat{m}_\ell^2 f_{+,0}^P(q^2)$ , we can understand that  $\mathcal{A}_P$ , shown in the figures 6 and 7, could be significantly affected by the charged Higgs couplings. However, the situation is different in the decays  $\bar{B} \rightarrow V\tau\bar{\nu}_\tau$ . Since the vector meson carries spin degrees of freedom, besides longitudinal parts which are similar to  $\bar{B} \rightarrow P\tau\bar{\nu}_\tau$ , there also exist transverse contributions. Therefore, the effects  $\propto \hat{m}_\ell^2 f_0^{P2}(q^2)$  become relatively small. This is the reason why the results in figures 8, 9





**Figure 9:** Legend is the same as figure 6 but for  $\bar{B} \rightarrow \rho^+ \tau \bar{\nu}_\tau$ .



**Figure 10:** Legend is the same as figure 6 but for  $\bar{B} \rightarrow D^{*+} \tau \bar{\nu}_\tau$ .

and 10 are not sensitive to the charged Higgs effects. We conclude that the charged Higgs contributions on  $\bar{B} \rightarrow V \ell \bar{\nu}_\ell$  are much less than those on  $\bar{B} \rightarrow P \ell \bar{\nu}_\ell$ .

## 5. Conclusion

Motivated by the recent measurement on the decay branching ratio of  $B^- \rightarrow \tau \bar{\nu}_\tau$ , we have studied the exclusive semileptonic decays of  $\bar{B} \rightarrow (\pi, D, \rho, D^*)^+ \ell \bar{\nu}_\ell$  in the MSSM. In particular, we have examined the charged Higgs effects from nonholomorphic terms at the large value of  $\tan \beta$ . To extract new physics contributions, we have defined several physical quantities to reduce uncertainties from the QCD as well as the CKM elements. Explicitly, for the allowed region of the charged Higgs mass, with the constraints from  $BR(B^- \rightarrow \tau \bar{\nu}_\tau)$  we have shown that  $\mathcal{A}(B^- \rightarrow \tau \bar{\nu}_\tau)$  and  $\mathcal{D}_{\pi, D} \sim 10\%$  are still allowed, whereas  $\mathcal{D}_{\rho, D^*}$  are small. Moreover, we have demonstrated that the angular asymmetries of  $\mathcal{A}_{\pi, D}$  could be significantly enhanced in the light  $M_{H^+}$  region, whereas those of  $\mathcal{A}_{\rho, D^*}$  are insensitive to the charged Higgs contributions. It is clear that if one of the above physical quantities is observed, it is a signature of new physics, such as the charged Higgs.

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## References

- [1] HEAVY FLAVOR AVERAGING GROUP (HFAG) collaboration, E. Barberio et al., *Averages of b-hadron properties at the end of 2005*, hep-ex/0603003.
- [2] K. Ikado et al., *Evidence of the purely leptonic decay  $B^- \rightarrow \tau \bar{\nu}_\tau$* , hep-ex/0604018; talk presented by T. Browder at ICHEP06, Moscow (2006)
- [3] BABAR collaboration, B. Aubert, *A search for  $B^+ \rightarrow \tau^+ \nu$  recoiling against  $B^- \rightarrow D^0 \ell^- \bar{\nu} X$* , hep-ex/0608019; see also talk presented by S.J. Sekula at ICHEP06, Moscow
- [4] R. Barlow, talk presented at ICHEP06, Moscow (2006)
- [5] PARTICLE DATA GROUP collaboration, S. Eidelman et al., *Review of particle physics*, Phys. Lett. **B 592** (2004) 1.
- [6] JLQCD collaboration, S. Aoki et al.,  *$B^0 - \bar{B}^0$  mixing in unquenched lattice QCD*, Phys. Rev. Lett. **91** (2003) 212001 [hep-ph/0307039]; HPQCD collaboration, A. Gray et al., *The B meson decay constant from unquenched lattice QCD*, Phys. Rev. Lett. **95** (2005) 212001 [hep-lat/0507015].
- [7] UTFIT collaboration, M. Bona et al., *The unitarity triangle fit in the standard model and hadronic parameters from lattice QCD: a reappraisal after the measurements of  $\Delta m_s$  and  $BR(B \rightarrow \tau \nu_\tau)$* , hep-ph/0606167.
- [8] W.-S. Hou, *Enhanced charged Higgs boson effects in  $B \rightarrow \tau \bar{\tau}, \mu \bar{\mu}$  and  $B \rightarrow \tau \bar{\tau}^+ X$* , Phys. Rev. **D 48** (1993) 2342.
- [9] A.G. Akeroyd and S. Recksiegel, *R parity violating enhancement of  $B_u^+ \rightarrow l^+ \nu$  and  $B_c^+ \rightarrow l^+ \nu$* , Phys. Lett. **B 541** (2002) 121 [hep-ph/0205176];

- [10] P. Krawczyk and S. Pokorski, *Strongly coupled charged scalar in B and T decays*, *Phys. Rev. Lett.* **60** (1988) 182;  
 J. Kalinowski, *Semileptonic decays of B mesons into  $\tau\bar{\tau}$  in a two Higgs doublet model*, *Phys. Lett.* **B 245** (1990) 201;  
 G. Isidori, *Limits to the charged Higgs sector from  $B \rightarrow X\tau\nu$* , *Phys. Lett.* **B 298** (1993) 409;  
 Y. Grossman and Z. Ligeti, *The inclusive  $B^- \rightarrow \tau\bar{\nu}X$  decay in two Higgs doublet models*, *Phys. Lett.* **B 332** (1994) 373 [[hep-ph/9403376](#)];  
 Y. Grossman, H.E. Haber and Y. Nir, *QCD corrections to charged Higgs mediated  $b \rightarrow c\tau\nu$  decay*, *Phys. Lett.* **B 357** (1995) 630 [[hep-ph/9507213](#)];  
 J.A. Coarasa, R.A. Jimenez and J. Sola, *The  $\tan\beta - M_{H^\pm}$  bound from inclusive semi-tauonic b decays in the MSSM*, *Phys. Lett.* **B 406** (1997) 337 [[hep-ph/9701392](#)].
- [11] B. Grzadkowski and W.-S. Hou, *Searching for  $B \rightarrow D\tau\bar{\nu}_\tau$  at the 10- percent level*, *Phys. Lett.* **B 283** (1992) 427;  
 M. Tanaka, *Charged Higgs effects on exclusive semitauonic b decays*, *Z. Physik C* **67** (1995) 321 [[hep-ph/9411405](#)];  
 K. Kiers and A. Soni, *Improving constraints on  $\tan\beta/m_H$  using  $B \rightarrow D\tau\bar{\nu}$* , *Phys. Rev. D* **56** (1997) 5786 [[hep-ph/9706337](#)].
- [12] T. Banks, *Supersymmetry and the quark mass matrix*, *Nucl. Phys.* **B 303** (1988) 172;  
 R. Hempfling, *Yukawa coupling unification with supersymmetric threshold corrections*, *Phys. Rev. D* **49** (1994) 6168;  
 C. Hamzaoui, M. Pospelov and M. Toharia, *Higgs-mediated FCNC in supersymmetric models with large  $\tan\beta$* , *Phys. Rev. D* **59** (1999) 095005 [[hep-ph/9807350](#)];  
 K.S. Babu, B. Dutta and R.N. Mohapatra, *Partial Yukawa unification and a supersymmetric origin of flavor mixing*, *Phys. Rev. D* **60** (1999) 095004 [[hep-ph/9812421](#)];  
 F. Borzumati, G.R. Farrar, N. Polonsky and S.D. Thomas, *Soft Yukawa couplings in supersymmetric theories*, *Nucl. Phys.* **B 555** (1999) 53 [[hep-ph/9902443](#)].
- [13] L.J. Hall, R. Rattazzi and U. Sarid, *The top quark mass in supersymmetric SO(10) unification*, *Phys. Rev. D* **50** (1994) 7048 [[hep-ph/9306309](#)].
- [14] M. Carena, S. Mrenna and C.E.M. Wagner, *MSSM Higgs boson phenomenology at the Tevatron collider*, *Phys. Rev. D* **60** (1999) 075010 [[hep-ph/9808312](#)];  
 K.S. Babu and C.F. Kolda, *Signatures of supersymmetry and Yukawa unification in Higgs decays*, *Phys. Lett.* **B 451** (1999) 77 [[hep-ph/9811308](#)].
- [15] K.S. Babu and C.F. Kolda, *Higgs-mediated  $B^0 \rightarrow \mu^+\mu^-$  in minimal supersymmetry*, *Phys. Rev. Lett.* **84** (2000) 228 [[hep-ph/9909476](#)].
- [16] G. Isidori and A. Retico,  *$B_{s,d} \rightarrow \ell^+\ell^-$  and  $K_L \rightarrow \ell^+\ell^-$  in SUSY models with non- minimal sources of flavour mixing*, *JHEP* **09** (2002) 063 [[hep-ph/0208159](#)].
- [17] G. D'Ambrosio, G.F. Giudice, G. Isidori and A. Strumia, *Minimal flavour violation: an effective field theory approach*, *Nucl. Phys.* **B 645** (2002) 155 [[hep-ph/0207036](#)];  
 A.J. Buras, P.H. Chankowski, J. Rosiek and L. Slawianowska, *Correlation between  $\Delta M_s$  and  $B_{s,d}^0 \rightarrow \mu^+\mu^-$  in supersymmetry at large  $\tan\beta$* , *Phys. Lett.* **B 546** (2002) 96 [[hep-ph/0207241](#)];  $\Delta M_{d,s}$ ,  $B_{s,d}^0 \rightarrow \mu^+\mu^-$  and  $b \rightarrow X_s\gamma$  in supersymmetry at large  $\tan\beta$ , *Nucl. Phys.* **B 659** (2003) 3 [[hep-ph/0210145](#)];  
 A. Dedes and A. Pilaftsis, *Resummed effective lagrangian for Higgs-mediated FCNC interactions in the CP-violating MSSM*, *Phys. Rev. D* **67** (2003) 015012 [[hep-ph/0209306](#)];

- [18] G. Isidori and P. Paradisi, *Hints of large  $\tan\beta$  in flavour physics*, *Phys. Lett.* **B 639** (2006) 499 [[hep-ph/0605012](#)]; *Higgs-mediated  $K \rightarrow \pi\nu\bar{\nu}$  in the MSSM at large  $\tan\beta$* , *Phys. Rev.* **D 73** (2006) 055017 [[hep-ph/0601094](#)];  
P. Paradisi, *Higgs-mediated  $e \rightarrow \mu$  transitions in II Higgs doublet model and supersymmetry*, *JHEP* **08** (2006) 047 [[hep-ph/0601100](#)]; *Higgs-mediated  $\tau \rightarrow \mu$  and  $\tau \rightarrow e$  transitions in II Higgs doublet model and supersymmetry*, *JHEP* **02** (2006) 050 [[hep-ph/0508054](#)]; *Constraints on SUSY lepton flavour violation by rare processes*, *JHEP* **10** (2005) 006 [[hep-ph/0505046](#)].
- [19] J.F. Gunion et al. , *The Higgs Hunter's Guide*, Perseus Publishing, 1990.
- [20] HPQCD collaboration, A. Gray et al., *The  $b$  meson decay constant from unquenched lattice QCD*, *Phys. Rev. Lett.* **95** (2005) 212001 [[hep-lat/0507015](#)].
- [21] H.-Y. Cheng, C.-K. Chua and C.-W. Hwang, *Covariant light-front approach for  $s$ -wave and  $p$ -wave mesons: its application to decay constants and form factors*, *Phys. Rev.* **D 69** (2004) 074025 [[hep-ph/0310359](#)].
- [22] P. Ball and R. Zwicky, *New results on  $B \rightarrow \pi, K, \eta$  decay formfactors from light-cone sum rules*, *Phys. Rev.* **D 71** (2005) 014015 [[hep-ph/0406232](#)].
- [23] C.-H. Chen and C.-Q. Geng, *Lepton angular asymmetries in semileptonic charmful  $B$  decays*, *Phys. Rev.* **D 71** (2005) 077501 [[hep-ph/0503123](#)].